ON STRIPLINE FOUR-PORT CIRCULATOR*

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Abstract

A rigorous and exact theory of four-port stripline circulator based on Green's function method is presented in this paper. Three circulation conditions were derived and numerical results are presented which can be used for practical four-port circulator design. Since the solution derived is exact, metal pin tuning is not necessary to fabricate the four-port circulator described in this paper.

Introduction

Most practical four-port circulators at the present time are realized by cascading two three-port circulators. The first single junction four-port circulator was fabricated by Yoshida et al.[1] Fay and Comstock[2] suggested the idea of operating a four-port circulator by using the n=0 and n=±1 modes. More recently, Fay and Dean[3] presented an alternative method of operation by using the n=±1 and n=2 modes in the circulator resonant cavity. However, only phenomenalogical description by the combination of various modes were given and most of them require the application of metal-pin tuning. Neither analytic design theory study nor quantitative data is available at present for the practical design of a four-port stripline circulator. In the present paper, we solve the four-port circulator problem rigorously by using Bosma's Green function method.[4,5] Numerical results are presented in this paper to facilitate the practical design of the stripline four-port circulators.

Theory

The structure of a four-port stripline circulator is shown in Figure 1. The center conductor is a disk resonator with radius R connected by four ninty-degree-separated striplines. The coupling angle of each stripline is denoted by 2Ψ as shown in Figure 1. This center conductor is sandwiched by two ferrite pucks with the same radius r=R. Outside the ferrite puck is a dielectric substrate with dielectric constant $\varepsilon=\varepsilon_d$. The two ferrite pucks have a tensor permeability $\bar{\bar{\mu}}$ and dielectric constant ε_f. The first step to solve this problem is to obtain Bosma's Green Function[4], which is given by

$$G(r,\theta;R,\theta') = -\frac{j\,Z_{eff}\,J_o(sr)}{2\pi\,J_o'(sR)} + \frac{Z_{eff}}{\pi}\sum_{n=1}^{\infty}\frac{\frac{k}{\mu}\frac{nJ_n(sR)}{(sR)}\sin n(\theta-\theta') - jJ_n'(sR)\cos n(\theta-\theta')}{\left(J_n'(sR)\right)^2-\left(\frac{k}{\mu}\frac{nJ_n(sR)}{(sR)}\right)^2}J_n(sr) \quad (1)$$

where μ, k = Polder tensor elements of the ferrite[6]

$Z_{eff} = 120\pi\cdot\sqrt{\frac{\mu_{eff}}{\varepsilon_f}}$ = effective intrinsic impedance of ferrite

$\mu_{eff} = \frac{\mu^2-k^2}{\mu}$ = effective permeability of ferrite

$s = \frac{\omega}{c}\sqrt{\mu_{eff}\varepsilon_f}$ = radial propagation constant of EM wave in ferrite

$J_n(sR)$ = Bessel Function of the first kind of the nth order

$J_n'(sR)$ = Derivative of $J_n(sR)$ with respect to the argument (sR).

This Green Function gives the electric field E_z at any point (r,θ) inside the ferrite puck caused by a unit magnetic field source $H_\theta = \delta(\theta-\theta')$ at the boundary point (R,θ'). Once the Green Function and the magnetic field are known, the electric field can be calculated by applying the following superposition integral

$$E_z(\theta) = \int_{-\pi}^{\pi} G(\theta,\theta')\,H_\theta(\theta')d\theta' \quad (2)$$

To calculate the magnetic field distribution, H_θ, some assumptions are needed. The first assumption we will make is that the wave propagating in the stripline is a TEM mode which has only two components, E_z and H_x. The second assumption we impose is that the arc effect at the junction is neglected. Based on these assumptions, we impose the following boundary conditions for the four-port circulator.

$$H_\theta(\theta') = \begin{cases} a \cdot & -\Psi<\theta<\Psi \\ b \cdot & \frac{\pi}{2}-\Psi<\theta<\frac{\pi}{2}+\Psi \\ c \cdot & \pi-\Psi<\theta<\pi+\Psi \\ d \cdot & \frac{3\pi}{2}-\Psi<\theta<\frac{3\pi}{2}+\Psi \\ 0 \cdot & \text{elsewhere, because of no radial currents} \end{cases} \quad (3)$$

Using (1) and (3) and noting that the wave impedances should be matched at the four junctions, the following three circulation conditions are derived after considerable algebraic calculations.

$$\text{(a)}\quad R^2 + S^2 = Q^2 \quad (4)$$

$$\text{(b)}\quad R^2 - S^2 = -PQ \quad (5)$$

$$\text{(c)}\quad S(Q-P) = RZ_d \quad (6)$$

where

$$P=\left(\frac{Z_{eff}}{\pi}\right)\left(\frac{\Psi B_o}{A_o}\right)+\sum_{n=1}^{\infty}\left(\frac{2\sin^2 n\Psi}{n^2\Psi}\right)\left(\frac{Z_{eff}}{\pi}\right)\frac{A_nB_n}{\left[A_n^2-\left(\frac{k}{\mu}\right)^2\left(\frac{nB_n}{sR}\right)^2\right]} \quad (7a)$$

$$Q=\left(\frac{Z_{eff}}{\pi}\right)\left(\frac{\Psi B_o}{A_o}\right)+\sum_{n=1}^{\infty}\left(\frac{2\sin^2 n\Psi}{n^2\Psi}\right)\left(\frac{Z_{eff}}{\pi}\right)\frac{A_nB_n\cos n\pi}{\left[A_n^2-\left(\frac{k}{\mu}\right)^2\left(\frac{nB_n}{sR}\right)^2\right]} \quad (7b)$$

$$R=\sum_{n=1}^{\infty}\frac{2\sin^2 n\Psi}{n^2\Psi}\left(\frac{Z_{eff}}{\pi}\right)\frac{\left(\frac{k}{\mu}\right)\left(\frac{nB_n}{sR}\right)^2\sin\left(\frac{n\pi}{2}\right)}{\left[A_n^2-\left(\frac{k}{\mu}\right)^2\left(\frac{nB_n}{sR}\right)^2\right]} \quad (7c)$$

*This research was supported by the U.S. Air Force Rome Air Development Center Post-Doctoral Program under Contract F30602-72-0497 and NSF Grant NO.GK31012X.

$$S=\left(\frac{Z_{eff}}{\pi}\right)\left(\frac{\Psi B_o}{A_o}\right)+\sum_{n=1}^{\infty}\left(\frac{2\sin^2 n\Psi}{n^2\Psi}\right)\left(\frac{Z_{eff}}{\pi}\right)\frac{A_n B_n \cos\left(\frac{n\pi}{2}\right)}{\left[A_n^2-\left(\frac{k}{\mu}\right)^2\left(\frac{nB_n}{sR}\right)^2\right]} \quad (7d)$$

$$A_n = J_n'(sR) \quad (7e)$$

$$B_n = J_n(sR) \quad (7f)$$

Output Wave Amplitude and Phase

If the three circulation conditions are satisfied, the field components at ports #3 and #4 will vanish. In other words, c=d=0 when circulation conditions are met. Under the same conditions, the field component at the output port #2 is given by

$$b = \frac{(-S + jR)}{Q} a \quad . \quad (8)$$

The wave magnitude of port #2 is equal to that of the input port, since

$$|b| = \frac{|-S+jR|}{|Q|}|a| = \frac{\sqrt{R^2+S^2}}{|Q|}|a| = \frac{\sqrt{Q^2}}{|Q|}|a| = |a|. \quad (9)$$

The phase of port #2, however, is shifted by an angle equal to

$$\theta = \tan^{-1}\left|\frac{R}{S}\right|. \quad (10)$$

Computation

The three circulation conditions in Equations (4) and (6) contain the three design variables for a four-port circulator. These three variables are the circulator disk radius, R, the ferrite anisotropic splitting, $|k/\mu|$, and the ferrite-dielectric impedance ratio, $|Z_{eff}/Z_d|$.

Figure 2 shows the calculated results of condition (a) by collecting terms up to the fourth order. The horizontal scale is the ferrite anisotropic splitting and the vertical scale is the Bessel function argument (equal to the radial propagation constant and radius product). The roots are searched only for those argument $X\leq 5$.

Figure 3 presents the roots of circulation condition (b). Since both conditions (a) and (b) must be satisfied simultaneously for the four-port junction to work as a circulator, therefore, we have to overlap condition (a) curves (Figure 2) to those of the condition (b) (Figure 3) and find out the intersection point. Only at the discrete intersection point, the junction works as a perfect four-port circulator. The coupling angle used in calculating Figures 2 and 3 is $\Psi=0.4$ radians. Only one intersection point A, marked by a small circle on both Figures 2 and 3 is found. This point A is located at sR=3.706 (condition (a)), with ferrite anisotropic splitting $k/\mu= -0.309$ (condition (b)), and impedance ratio $|Z_{eff}/Z_d|=0.300$ (condition (c)). The operating point A is studied in more detail to show its dependence on various coupling angles.

Figure 4 shows the extended results of the disk radius as a function of stripline coupling angles. Obviously, the radius of the ferrite puck decreases with increasing coupling angles. The radius of a four-port stripline circulator is therefore much larger than that of a three-port circulator whose radius is usually given by sR=1.84[5].

Figure 5 shows the extended results of the second circulation condition which determines the ferrite magnetization as a function of coupling angles. For small coupling angles ($\Psi<0.3$), the ferrite anisotropic splitting decreases with increasing coupling angle magnitude. However, for large coupling angles, this splitting $|k/\mu|$ increases with increasing coupling angles.

Figure 6 indicates the variation of ferrite-dielectric intrinsic impedance ratio as a function of coupling angles. It shows that with a coupling angle of $\Psi=0.1$, the impedance ratio is approximately unity. However, when the coupling becomes tighter, this impedance ratio drops to 0.3. The phase shift of the output wave of this four-port circulator is calculated to be 90° with respect to the input wave.

In addition to point A, four other points, B, C, D, and E, indicated by small triangles, are possible for specific coupling angles. However, at $\Psi=0.4$, they are actually not intersection points but only closely tangent to each other between curves in Figures 2 and 3. Calculated results show that point B does give solutions when $0.1<\Psi<0.3$ and point C gives solutions at $\Psi=0.6$.

Conclusions

Three circulation conditions are presented in this paper to fabricate a four-port circulator. Numerical results of these three conditions are also calculated and can be used to choose the size of ferrite disk, ferrite anisotropic splitting, as well as the dielectric constant of the outside substrate.

Although Fay and Comstock[2] suggested to operate a four-port circulator by using the n=0 and n=±1 modes, due to the presence of n=2 mode which has a resonant root between that of the n=0 and n=1 resonances[5] (unless a capacitive metal pin is used), it is very difficult to operate a X-circulator by using only the n=0 and n=±1 modes without the effect of the n=2 modes. Point A in Figure 2 is operated at a location where n=0, ±3, and ±2 modes appear. All these modes work together and make a perfect four-port circulator.

References

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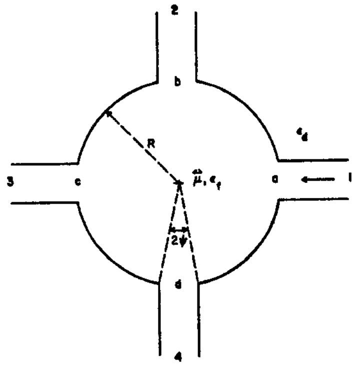

FIG. 1 STRUCTURE OF THE FOUR-PORT CIRCULATOR

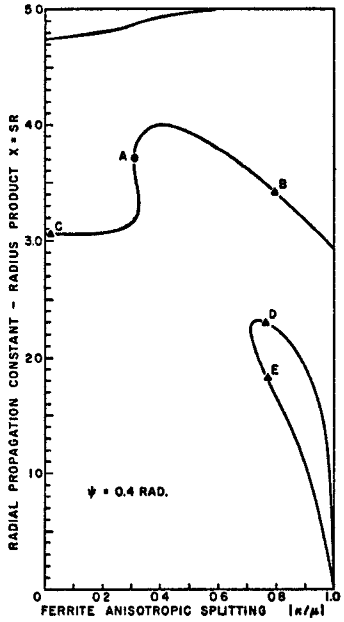

FIG. 2 CALCULATED RESULTS OF THE FIRST CIRCULATION CONDITION OF A STRIPLINE FOUR-PORT CIRCULATOR

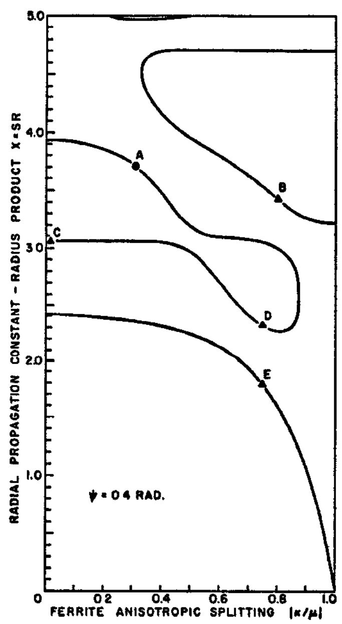

FIG. 3 CALCULATED RESULTS OF THE SECOND CIRCULATION CONDITION OF A STRIPLINE FOUR-PORT CIRCULATOR

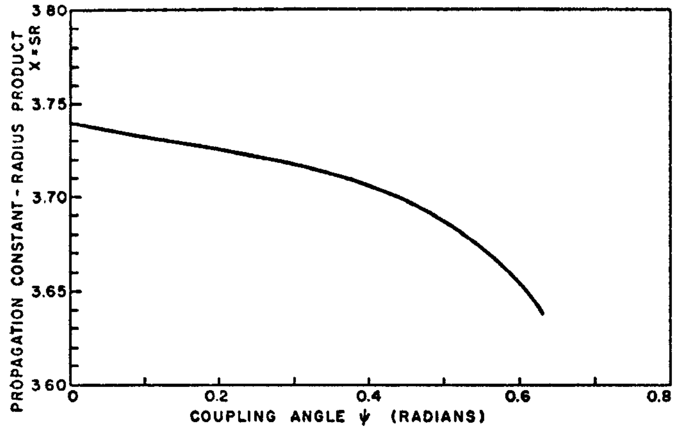

FIG. 4 EXTENDED FIRST CIRCULATION CONDITION OF A FOUR-PORT STRIPLINE CIRCULATOR AS A FUNCTION OF COUPLING ANGLES

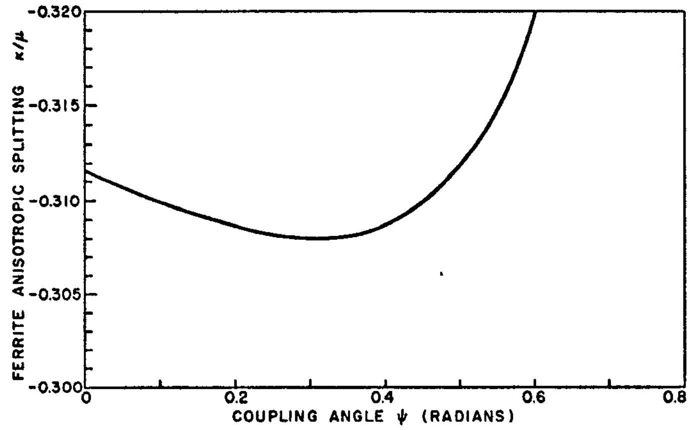

FIG. 5 EXTENDED SECOND CIRCULATION CONDITION OF A FOUR-PORT CIRCULATOR AS A FUNCTION OF COUPLING ANGLES

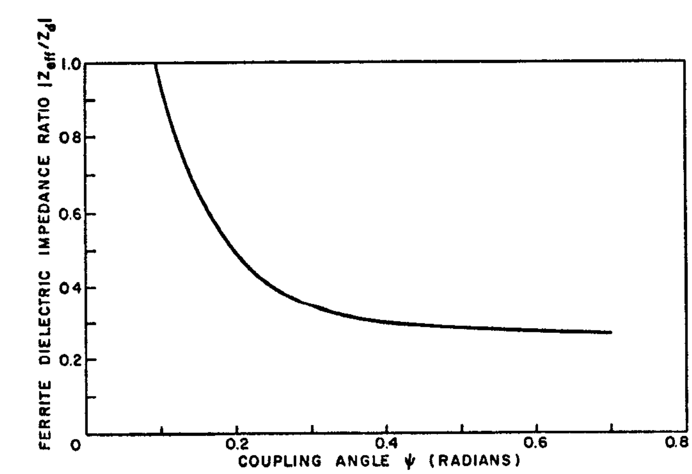

FIG. 6 EXTENDED THIRD CIRCULATION CONDITION OF A FOUR-PORT CIRCULATOR AS A FUNCTION OF COUPLING ANGLES